M.Sc. (Mathematics) Sem. III

ADVANCED TOPOLOGY

Locally compact space : Let X be a topological space. Let $x \in X$ be a point. Then X is called locally compact at the point $x \in X$ if x has a compact neighbourhood in X.

If X is locally compact at all points of X then X is called a locally compact space.

In other words, X is locally compact at a point x if given any neighbourhood N of x there exists a compact neighbourhood M of x such that $M \subset N$.

Some important properties

1. If a space X is regular and locally compact at a point $x \in X$ then x has a local base consisting of compact neighbourhoods.

PTO

2. If a Hausdorff space X is locally compact at a point $x \in X$ then the family of compact neighbourhoods of x is a local base at x.

3. Every locally compact Hausdorff space is regular.

4. Every locally compact Hausdorff space is completely regular.

5. A locally compact Hausdorff space is not necessarily normal.

6. Every compact space is locally compact.

7. All discrete spaces are locally compact.

8. The real line is locally compact.

9. The space of rationals with usual topology is not locally compact at any point.

10. Local compactness is not a ~~hereditary~~ hereditary property.
It is weakly hereditary.